Unveiling new analytical solutions to the Dirac equation

Andre G. Campos
Max Planck Institute for Nuclear Physics, Heidelberg, Germany

The complexity of the structure of the Dirac equation, which is a system of four coupled differential equations, renders it very difficult to study. The number of closed-form solutions is very limited due to the intricate structure of Dirac matrices which couple the components of the four-spinor wave function. For this reason, only highly symmetric systems can be studied by analytical means [1]; the mathematical description of more realistic systems should be based on approximation methods such as semi-classical theory [2] or numerical calculations [3–5]. However, the typical time scale of the electron dynamics is usually much smaller than the time scales of interesting phenomena, rendering the numerical solution notoriously difficult and requiring substantial computer resources [6]. Here, we provide a very powerful method to arrive at the sought-after general analytical solutions to the Dirac equation. For instance, we construct time-dependent electromagnetic fields that coherently steer the electron’s four-spinor wave function to follow a given path. Moreover, we present new and nontrivial analytical solutions to the Dirac equation that are unique to relativistic quantum systems. The newly developed solutions unravel exciting new insights on the complex quantum dynamics of relativistic electrons. Our method constitutes an important tool with very broad range of applications.