The classical dynamics of a conservative system is time-reversible. If we watch a movie backwards in the absence of friction, it will show dynamics perfectly consistent with the laws of motion, so we may not distinguish whether we watch the movie forward or backwards in time from the dynamics alone. However, when the system has more than a few degrees of freedom—such as during the starting break in a game of pool—then the likelihood that the evolution is either forward or backward in time may differ, so it becomes possible to distinguish an arrow of time statistically.

The quantum dynamics of a conservative and unmeasured system is similarly time-reversible. For example, the Schrödinger equation becomes invariant under time-inversion if the position-space wavefunction is complex-conjugated.

The introduction of a sequence of measurements seems to break such dynamical symmetry, however, for two distinct reasons. First, obtaining definite measurement results traditionally collapses the wavefunction, which produces non-unitary evolution that is distinct from the Schrödinger equation and not reversed by the same operation. Second, the randomness of each measurement creates an intrinsic asymmetry between an unknown future and a definite past. These reasons have contributed to the view that quantum mechanics is fundamentally asymmetric in time.

We seek to clarify this apparent discrepancy between classical and quantum reversibility. We pose the time-reversibility problem for a continuous quantum measurement in the following way: Suppose we are given a movie of stochastic quantum state dynamics along with its associated noisy detector output (a sort of “soundtrack” for the movie). We are then asked to determine whether the movie shows the forward evolution of the state, or whether the movie has been reversed, as depicted in Fig. 1. In the simplest case of a monitored qubit, we find that such a movie played backwards obeys time-reversed equations of motion if we also flip the sign of its soundtrack (measurement record).

Despite this restoration of dynamical reversibility, a statistical arrow of time emerges, and may be quantified by the log-likelihood difference between forward and backward propagation hypotheses. We then show that such reversibility is a universal feature of non-projective measurements, with forward or backward Janus measurement sequences that are time-reversed inverses of each other [1].

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FIG. 1. A single quantum trajectory of a continuously monitored qubit. The $x$ and $z$ Bloch sphere coordinates of a qubit change due to both unitary and measurement dynamics. The red and blue colors denote positive and negative values of the $x$ coordinate. The boundary states are shown as green and red dots. Is time running forward with measurement record $r(t)$ or backward with flipped record $\tilde{r}(t)$?