Single and paired parametric oscillators

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Parametric resonance phenomena are important in several areas of physics, ranging from the design of parametric amplifier circuits to the recently introduced Quantum Amplification by Superradiant Emission of Radiation (QASER).\textsuperscript{[1][2]} The standard mathematical description underlying these effects is the Hill differential equation of a linear oscillator with a periodic frequency modulation. However, in the QASER we deal with two oscillators and an asymmetric periodically modulated coupling giving rise to a frequency-difference resonance.

Floquet theory yields elegant stability results for these models, and the two-oscillator case contains mathematical features not seen with the single oscillator, corresponding directly to the frequency-difference resonance phenomenon that is essential to the QASER.

We additionally consider the tractable case where the periodically varying coefficient is a $\delta$-function instead of a sinusoid and obtain analytical formulas for the gain. The mathematics is related to the Kronig–Penney model of condensed-matter physics.

Gain $g$ (left) and discriminant $\Delta$ (right) of the characteristic Floquet exponent corresponding to the set of coupled driven oscillator equations representing a model of the QASER as a function (blue lines) of the oscillator frequency $\omega_0$ for a coupling $\Omega = \omega_0/2$ and a fixed amplitude $q = 0.4$ of the drive. Whereas $g$ follows directly from numerical integration we obtain $\Delta$ from a Floquet analysis of the corresponding system of four first-order differential equations. We note that in the neighborhood of $\omega_0 = 8$ and $\omega_0 = 16$ the discriminant $\Delta$ assumes negative values, but positive ones around $\omega_0 = 1$. Orange lines follow from a version of the model where the cosine drive (inset) is replaced by a periodic sequence of $\delta$-function kicks.