Cooperative spontaneous emission of N atoms: collective non-local effects

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Many-body effects in spontaneous emission from N-atoms are a subject of current interest [1–4]. Here we investigate influence of non-local (memory) effects on evolution of collective atomic states (for details see [5]). We assume that atomic cloud has a shape of an infinitely long cylinder (or a slab) of radius (thickness) \(R\). For a dense cloud of volume \(V\) evolution of the atomic system is described by equation [4]

\[
\frac{\partial \beta(t, \mathbf{r})}{\partial t} = -\frac{V_{\text{ph}}}{(2\pi)^3 V} \int d\mathbf{k}' \int d\mathbf{r}' g_{\mathbf{k}' \mathbf{r}'} \int_0^t dt' \beta(t', \mathbf{r}') \times
\left[ e^{i(\nu_{k'} - \omega)(t' - t)} + e^{i(\nu_{k'} + \omega)(t' - t)} \right] \exp[i\mathbf{k}' \cdot (\mathbf{r} - \mathbf{r}')], \tag{1}
\]

where \(\beta(t, \mathbf{r})\) is the probability amplitude to find atom at position \(\mathbf{r}\) excited at time \(t\), \(V_{\text{ph}}\) is the photon volume, \(\omega\) is the atomic frequency and \(\nu_{k'} = ck'\). Eq. (1) is non-local, that is evolution of the system at time \(t\) depends on \(\beta(t, \mathbf{r})\) values in the previous time moments.

We found that cross-over between local and non-local behavior occurs at

\[
R \sim R_0 = \left( \frac{c V}{2\pi\gamma N} \right)^{1/4}, \tag{2}
\]

where \(\gamma\) is the single atom decay rate. For \(R \ll R_0\) the system evolution is local. In the local regime eigenfunctions of Eq. (1) have the form (for cylinder)

\[
\beta(t, \mathbf{r}) = J_n(\alpha \rho) e^{in\phi + ik_z z} e^{-\lambda t}.
\]

For \(k_z \geq k_0 = \omega/c\) the eigenvalues \(\lambda\) are purely imaginary. The corresponding eigenstates do not decay and yield no photon emission.

In contrast, in the non-local limit \((R \gg R_0)\) solution of Eq. (1) is

\[
\beta(t, \mathbf{r}) = \beta_0(\rho) e^{in\phi} e^{ik_0 z} \cos(\Omega t),
\]

where \(\Omega = \sqrt{2\pi\gamma c^3 N/V \omega^2}\) is the collective Rabi frequency which is proportional to the square root of atomic density \(N/V\) and analogous to plasma frequency in classical electrodynamics. In the non-local regime the probability to find atoms excited \(P(t)\) oscillates with frequency \(\Omega\) (see Fig. 1). Such oscillations indicate that photon is emitted and then reabsorbed.

For current experiments on collective excitation of \(^{57}\text{Fe}\) nuclei in solid-state samples [6] \(N/V = 10^{22}\) \(\text{cm}^{-3}\), \(\gamma = 7 \times 10^6\) \(\text{s}^{-1}\) and Eq. (2) gives \(R_0 \sim 160\) nm. Experiments of Ref. [6] are conducted for slab thickness \(\sim 1\) nm which corresponds to the local regime. By increasing thickness of the nuclear layer one can observe non-local behavior of collective many-particle emission.

FIG. 1: Probability \(P(t)\) to find atoms excited in local and non-local regimes for guiding states.