Suppression of the quantum-mechanical collapse by repulsive interactions in a quantum gas

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The quantum-mechanical collapse (alias fall onto the center of particles attracted by potential \(-U_0r^{-2}\)) is a well-known issue in the quantum theory [1]. It is closely related to the quantum anomaly, i.e., breaking of the scaling invariance of the respective Hamiltonian by the quantization. We demonstrate that the mean-field repulsive nonlinearity prevents the collapse and thus puts forward a solution to the quantum-collapse problem different from that previously developed in the framework of the linear quantum-field theory. This solution may be realized in the 3D or 2D gas of dipolar bosons attracted by a central charge, or in the 2D gas of magnetic dipoles attracted by a current filament. In the 3D setting, the dipole-dipole interactions are also taken into regard, in the mean-field approximation, which results in a redifinition of the scattering length which accounts for the contact repulsion between the bosons: 

\[ a_s \rightarrow a_s + md^2\hbar^2, \]

where \(m\) and \(d\) are the mass and dipole moment of the bosons.

The scaled form of the corresponding Gross-Pitaevskii equation, which also includes the external harmonic trap with strength \(\Omega^2\), is

\[ i\partial_t \psi = -(1/2)\left(\nabla^2 + U_0r^{-2} - \Omega^2r^2\right) + |\psi|^2\psi. \]

In lieu of the collapse, the cubic nonlinearity creates a 3D ground state (GS), which does not exist in the respective linear Schrödinger equation in the case of \(U_0 > 1/4\). For \(U_0 < 1/4\), when the Schrödinger equation still does not lead to the collapse, the cubic nonlinearity makes the GS wave function normalizable (with \(\Omega^2 = 0\)), on the contrary to the situation in the linear Schrödinger equation. In the same case, the addition of the harmonic trap \((\Omega^2 > 0)\) gives rise to a tristability.

In the 2D setting, the cubic nonlinearity is not strong enough to prevent the collapse; however, the quintic term does it, creating the GS, as well as its counterparts carrying the angular momentum (vorticity). Counter-intuitively, such self-trapped 2D modes exist even in the case of a weakly repulsive potential, \(+U_0r^{-2}\) (with \(\Omega^2 = 0\)). The 2D vortical modes avoid the phase singularity at the pivot \((r = 0)\) by having the amplitude diverging at \(r \rightarrow 0\), instead of the usual situation with the amplitude of the vortical mode vanishing at \(r \rightarrow 0\) (the norm of the mode converges in spite of the singularity of the amplitude at \(r \rightarrow 0\)). In the presence of the harmonic trap, the 2D quintic model with the weakly repulsive potential gives rise to three confined modes, the middle one being unstable, spontaneously transforming itself into a breather. In both the 3D and 2D cases, the GS wave functions are found in a numerical form, and also in the form of an analytical approximation, which is asymptotically exact in the limit of a large norm.