Problem of cooperative spontaneous emission of atomic ensembles is subject of long standing interest [1–3]. We study collective Lamb shift and decay rate of a single scalar photon stored in spheroidal (prolate or oblate) cloud of volume \( V = \frac{4\pi}{3}a^2b \) of \( N \) two-level atoms \((\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1)\). We find eigenstates of the system in Markovian approximation and analyze asymptotic solution in the small and large sample limits. General spheroidal geometry allows us to study evolution of eigenstates as the cloud changes shape from a sphere to a cylinder, slab or disc and make a comparison between the limiting geometries.

Cooperative spontaneous emission is an example of a many-body problem of \( N \) atom collectively interacting with electromagnetic field. It reduces to finding all eigenstates for probability amplitude \( \beta(r, t) = e^{-\lambda t} \beta(r) \) that obeys integral evolution equation with exponential kernel [4]

\[
\frac{\partial \beta(t, r)}{\partial t} = i\gamma N \int_V d\mathbf{r}' \frac{\exp(i k_0 |r - r'|)}{k_0 |r - r'|} \beta(r'),
\]

where \( \gamma \) is the single atom decay rate, \( k_0 = \omega/c \). FIG. 1 shows eigenvalues \( \lambda \) obtained by solution of Eq. (1) as the atomic cloud changes its shape.

![FIG. 1. Real part (cooperative decay rate) - [(a) and (c)], and imaginary part (collective Lamb shift)- [(b) and (d)] of dimensionless eigenvalue \( \tilde{\lambda}_{nl} = k_0 V \lambda_{nl} / 4\pi \gamma N \) as a function of \( k_0a \) (a < b in prolate and a > b in oblate geometry) for the spheroid of volume \( V \) for different values of flattening factor \( f = 1 - \min(a, b)/\max(a, b) \) that defines the shape of the cloud. \( f = 0 \) corresponds to sphere and \( f = 1 \) corresponds to infinite slab (oblate) and cylinder (prolate). Solid line corresponds to for spheroid [5], dash line is a solution for infinite cylinder [6], and dot line is a solution for sphere [4]. (e) and (f) are examples of oblate (a > b) and prolate (a < b) spheroids, respectively. (g) and (h) are real and imaginary parts of \( \alpha_{nl} = \sqrt{1 - i/\tilde{\lambda}_{nl}} \) for \( n = 0, l = 1 \) as the atomic cloud changes its shape from sphere \( f = 0 \) to infinite slab or cylinder \( f = 1 \) at fixed \( k_0a = 0.1 \) (solid line), 0.5 (dot-dash line), 5 (dash line), 10 (dot line).