In recent years there has been a great deal of progress towards cooling heteronuclear alkali metal dimer molecules to quantum degeneracy in their absolute ground state [1]. Ultracold polar molecules such as these are of great interest in many fields ranging from precision science and metrology to many-body physics [2]. Of particular interest to many-body physics is the fact that heteronuclear molecules have permanent electric dipole moments which give rise to long-range interactions in a DC electric field. Also, because of a nuclear quadrupole coupling, this dipole moment can also be used to access the many internal hyperfine states in a controlled way. This paves the way to rich, complex dynamics, such as the many-body dephasing of Rabi flopping between two internal states as shown in Fig. 1.

Much of the work regarding ultracold polar molecules in optical lattices has focused on the recreation of spin models using often complex experimental setups. Instead, we focus on the natural many-body physics of such molecules in setups similar to current experiments. To this end we present the Molecular Hubbard Hamiltonian (MHH) [3], which describes the low-energy physics of ultracold $^1\Sigma$ polar molecules trapped in an optical lattice in the presence of DC electric and magnetic fields and an AC microwave field. The electric field orients the dipole moments and thus determines the interactions, the magnetic field controls the nuclear spin states, and the AC field drives transitions between rotational states. Because multiple rotational states are populated, dipolar processes which exchange a rotational quantum between molecules are allowed and compete with direct interactions which do not change the internal state. We also present a thorough exposition of all energy scales down to 10Hz, which allows us to systematically truncate terms in the MHH to a consistent order of approximation. This controlled truncation is important when discussing the stability of many-body phases. In addition to discussing the derivation of the MHH, we will discuss its behavior in different field regimes and for different experimentally relevant species, and provide mappings to known spin models in specific cases in order to demonstrate the underlying many-body physics.