How to optimally estimate a physical parameter using light?

a practical answer

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Optical techniques are widely used to make accurate measurements because they can reach very high precision and sensitivity levels. It is therefore important to know what is the ultimate limit of sensitivity that can be possibly achieved in the estimation of a parameter given some constraints such as a fixed mean photon number N. Many studies have been devoted to finding ways to enhance as much as possible the sensitivity of parameter estimation using quantum resources, such as squeezed states, quadrature entangled states, Fock states, or NOON states.

The talk will envision the problem of optimized parameter estimation in a way that is closely related to experimental considerations. As all quantum limits scale as some inverse power on N, only *intense light* is worth considering from a practical point of view when one wants to reach ultra-high measurement sensitivities. States such as the NOON states have been so far produced for N values of order of 10 at best and are very sensitive to decoherence processes. On the other hand multimode Gaussian states such as squeezed or quadrature entangled states are presently available non classical states of light with N as high as $10^{16}$. This is the reason why we are interested in determining the ultimate sensitivity in parameter estimation using intense Gaussian light.

The originality of the present approach is its *multi-modal character*. A multimode quantum state is defined not only by the value of the coefficients of its decomposition on the Fock state basis but also on the spatio-temporal shape of the different modes on which these Fock states are defined. This leaves us two kinds of degrees of freedom on which to act. We will show that the ultimate sensitivity with Gaussian light is obtained not only by choosing the best possible Gaussian quantum state, but also by putting this state in the best possible mode basis.

We firstly derive the Quantum Cramer Rao limit for parameter estimation using multimode pure Gaussian states of light. We then exhibit an optical set-up that reaches such a limit, namely a homodyne detection scheme that uses an appropriately shaped mode as a local oscillator. We finally show that the most economical way to maximize the sensitivity is to put the most squeezed state available in a well-defined light mode, called the detection mode. One cannot take advantage of squeezed fluctuations or quantum correlations coming from different modes to improve the estimation of a single parameter. We therefore advise experimentalists to produce a single vacuum squeezed state, to put it in the detection mode, and to mix it with a coherent state of high mean photon number N. Doing that, they will be sure that nobody else will make a more sensitive estimation of the variation of the parameter of interest.

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