Non-local effects in single photon superradiance

Anatoly Svidzinsky

Department of Physics & Astronomy, Texas A&M University

Collective spontaneous emission from a cloud of N atoms has been a subject of long standing interest [1]. Recent calculations focus on the problem in which a single photon is stored in the cloud of atoms and then retrieved at a later time. Here I treat the problem fully quantum mechanically and take into account non-local (retardation) effects in photon emission. I found that for a weakly excited dense atomic cloud of volume $V$ atomic evolution is described by the integral equation

$$\frac{\partial \beta(t,r)}{\partial t} = i\gamma \frac{N}{V} \int dr' \frac{\exp(ik_0|r-r'|)}{k_0|r-r'|} \beta \left(t - \frac{|r-r'|}{c}, r'\right), \quad (1)$$

where $\beta(t,r)$ is the probability amplitude to find atom at position $r$ excited at time $t$, $\gamma$ is the single atom decay rate and $k_0=\omega/c$. For atomic slab of thickness $R \gg \lambda$ (Fig. 1a) I found an exact analytical solution of Eq. (1) describing system evolution after excitation by a plane wave $\beta(0, r) = e^{ik_0z}$

$$\beta(t, r) = e^{ik_0z} \left( \cos(\Omega t) + \Theta(ct-z) \frac{\Omega}{c} \int z' \sqrt{\frac{ct-z'}{z'}} J_1 \left( \frac{2\Omega}{c} \sqrt{z'(ct-z')} \right) dz' \right) \quad (2)$$

Fig. 1 compares solution (2) with those obtained omitting retardation. Non-local effects dramatically modify system evolution for thick atomic samples $R > c/\Omega$ leading to oscillations of atomic population with collective frequency $\Omega = \sqrt{N\lambda^2c\gamma/2\pi V}$ (Fig. 1c). Eq. (2) yields insight on how cross-over between local and non-local dynamics occurs. In particular, it shows that initially non-local evolution becomes local at large time. Also, in the transition region atomic excitation in some parts of the sample rises above its initial value. Cross-over between local and non-local behavior can be observed by increasing sample size or varying atomic density.