Collective nonlinear oscillators from 
two-level atoms

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Nonlinear interactions and chaotic phenomena are the manifestations of the complex dynamics in laser-matter interactions. Development of the complex nonlinear models led to a realistic laser description and the subsequent understanding of the nontrivial laser-matter interactions [1]. In spite of these remarkable successes the roots of these complex models have never been properly established from the fundamental light-matter interaction. Here we demonstrate that simple two-level interacting system described by Maxwell-Schrödinger equations provides a full basis for the description of a wide range of complex nonlinear phenomena. In particular we show that interaction of a space-independent electromagnetic pulse with a gas of two-level atoms naturally leads to the Van der Pol oscillator equation [2]. By restoring the propagation of the electromagnetic pulse we arrive at the Duffing oscillator equation [3]. Our result provides an important link between physics of the chaotic phenomena on the one hand and two-level treatment of laser-matter interaction on the other hand. We suggest new experimental realizations of the nonlinear light-matter interactions that can lead to a significant progress in fundamental physics, quantum optics and engineering [4]. 

\begin{itemize}
\item [1] M. Sargent, M. O. Scully, and W. E. Lamb, Jr., Laser Physics, WV Press, 1974 
\item [3] G. Duffing, Forced vibrations with a variable natural frequency, F. Vieweg, 1918. 
\end{itemize}

FIG. 1: Phase portrait of the Van der Pol oscillator.