The 2013 paper by A. Svidzinsky, L. Yuan, and M. O. Scully\(^1\) presents a novel theoretical mechanism for a quantum amplifier based on collective superradiant emission. If this mechanism were experimentally confirmed it could lead to, for example, the generation of XUV radiation from an infrared driving field and applications in standoff detection. There is no violation of energy conservation because the generated field is much weaker than the driving field. This proposed mechanism presumes an ingenious suppression of the time-dependent Stark shift which ordinarily arises from the driving field. With this proviso, it was argued that parametric resonance between the driving field and collective superradiant oscillations of the atomic polarization can yield light amplification at high frequencies.

Central to the derivation of this phenomenon is the function \(\Omega_s(t)\) describing the envelope of the superradiant pulse. With suitable assumptions and approximations, the time evolution of \(\Omega_s\) is given by:

\[
\ddot{\Omega}_s + \Omega_s \left[ 1 - \delta \cos(2vt) \right] \Omega_s = 0
\]

This linear second order homogeneous differential equation is known as the Mathieu equation. In my poster I present the possible solutions for this equation. There are periodic solutions for special values of the constants. A consequence of the Floquet theorem are solutions which tend to grow or decay exponentially. A connection is made between the Fourier and Bessel representations of the solutions. In solutions with exponential increase, the sensitivity of the gain to the fine-tuning of the driving frequency is explored.

Figure:
The behavior of solutions as a function of certain quantities derived from the equation parameters.\(^2\)

\(^2\)Reproduced from fig. 5.4, p. 563, P. M. Morse and H. Feshbach, Methods of Theoretical Physics, McGraw-Hill, New York, (1953).