Entanglement in transverse position and momentum is of growing interest as their conjugate nature underlies imaging and propagation. Spatial entanglement also evolves during propagation, from correlated in the near-field to anti-correlation in the far-field, which is used to demonstrate the EPR paradox [1,2]. Between the two, the correlation in the positions of the photons disappears and the entanglement migrates into the phase of the biphoton wave function $\psi(x_1, x_2)$ [3,4].

Fig. 1. Evolution of spatial entanglement with propagation. Entanglement (non-separability) initially (left) resides in (top) amplitude, migrates into phase (middle) phase upon propagation (center), and then back to amplitude in the (right) far-field. Entanglement migration is quantified by (bottom) amplitude and phase components of Schmidt number versus propagation distance.

Here, we develop an analytic model of the evolution of spatial entanglement upon propagation in terms of the Schmidt number $K$. For the common double-Gaussian model of biphoton wave functions, the Schmidt number may be expressed as a sum of the degree of entanglement in amplitude, $K_{amp}$, and in phase, $K_{phase}$. For spatially entangled photon pairs, such as those generated from spontaneous parametric down-conversion, entanglement resides in the wave function’s amplitude in the near-field, migrates into the phase upon propagation, and then back again into the amplitude in the far-field (Fig. 1) [5]. We extend this model to incorporate effects of the pump geometry and higher spatial frequency content of more realistic non-Gaussian biphoton wave functions. To do so, we borrow the concept of a beam quality parameter, $M^2$, from classical laser physics and apply it to biphotons. The increased spatial frequency content leads to more rapid diffraction, which may now be accounted for in terms of “biphoton quality parameters” $M^2$ in the sum and difference of coordinates of the individual photons. Nominal unity values represent ideal wave functions, while those with $M^2 > 1$ have more rapid decay of spatial correlation, and migration of entanglement into phase, upon propagation.

Fig. 2. (top row) Upon propagation, spatial correlation in the near-field decreases and anti-correlation increases towards the far-field. (bottom) Measured (circles) amplitude entanglement compared with (solid curve) fit for $M^2 = 2.7$, and (dashed curve) $M^2 = 1$ for comparison versus propagation distance on a logarithmic scale. [5]

Experimentally, we measure the spatial correlation for many defocused planes about the near- and far-fields by imaging biphotons with a single-photon sensitive electron multiplying CCD in the photon counting regime [5]. Spatial correlations are calculated for each position and the degree of entanglement in amplitude ($K_{amp}$) is calculated. Incorporation of $M^2$ into our model allows fitting of the fall off of spatial correlation in the near field and rise of anti-correlation in the far-field (Fig. 2). We see a much more rapid fall off of entanglement in amplitude in the form of spatial correlation upon propagation, and more rapid migration back to in the form of anti-correlation, than for the ideal Gaussian case ($M^2 = 1$). Prospects of measuring phase entanglement via interferometric methods will be discussed.

References